A Mass Conservative Mesoscopic Drying Model

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INSTITUT DES SCIENCES ET INDUSTRIES DU VIVANT ET DE L'ENVIRONNEMENT
PARIS INSTITUTE OF TECHNOLOGY FOR LIFE, FOOD AND ENVIRONMENTAL SCIENCES

Overview of presentation

1. Structure of softwood
2. Computational Mesoscopic Drying Model
   - Transport equations
   - Mesh generation
   - Spatial discretisation
   - Novel strategy: Jacobian-free Exponential Euler Method (EEM)

2D board cross-sections

3. Mass-conservative formulation
   - Partitions the moisture content field over the individual sub-control volumes surrounding each node within the mesh
4. Results: Mass-conservative versus Existing formulation
5. Conclusions

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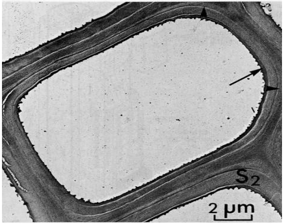
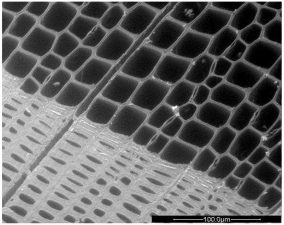


Earlywood Tracheid[1]



Electron Microscope Image

[1] J. F. Siau (1984)

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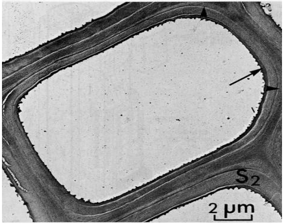

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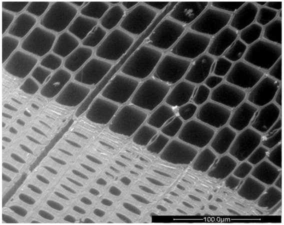

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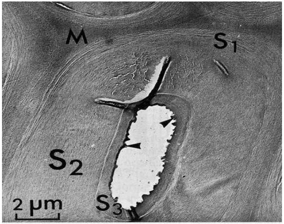

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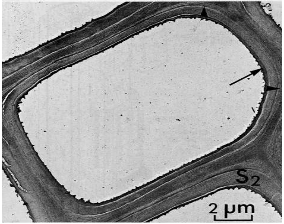

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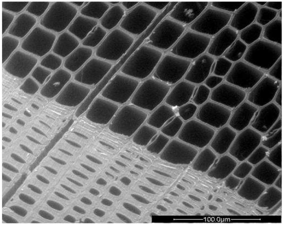

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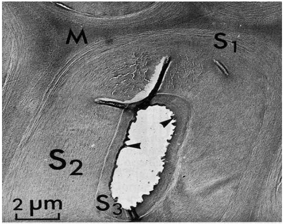

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Observable scale

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Transport equations

Mesoscopic Drying Model: TransPore (State Variables: X, T and $\overline{\rho}_{\mathrm{a}} = \epsilon_{\mathrm{g}}\rho_{\mathrm{a}}$)[2]

- *Liquid conservation*

$$\frac{\partial}{\partial t}\Big(\rho_0(\boldsymbol{x})X + \epsilon_{\mathrm{g}}\rho_{\mathrm{v}}\Big) + \nabla\cdot\Big(\rho_{\mathrm{w}}\boldsymbol{v}_{\mathrm{w}} + \rho_{\mathrm{v}}\boldsymbol{v}_{\mathrm{g}} + \overline{\rho_{\mathrm{b}}\boldsymbol{v}_{\mathrm{b}}}\Big) = \nabla\cdot\Big(\rho_g\mathbf{D}_{\mathrm{eff}}\nabla\omega_{\mathrm{v}}\Big)$$

- *Air conservation*

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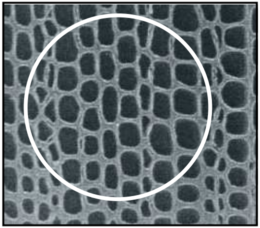

Representative Elementary Volume

- *Energy conservation*

$$\frac{\partial}{\partial t}\Big(\rho_0(\boldsymbol{x})\big(Xh_{\mathrm{w}} + h_{\mathrm{s}}\big) + \epsilon_{\mathrm{g}}\big(\rho_{\mathrm{v}}h_{\mathrm{v}} + \rho_{\mathrm{a}}h_{\mathrm{a}}\big) - \int_0^{\overline{\rho}_{\mathrm{b}}}\Delta h_{\mathrm{w}}\,d\rho - \epsilon_{\mathrm{g}}P_{\mathrm{g}}\Big)$$
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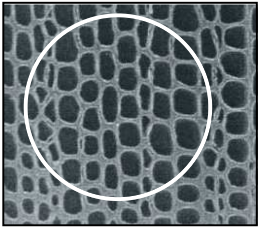

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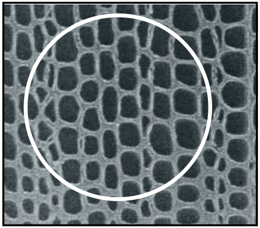

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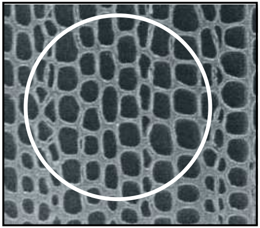

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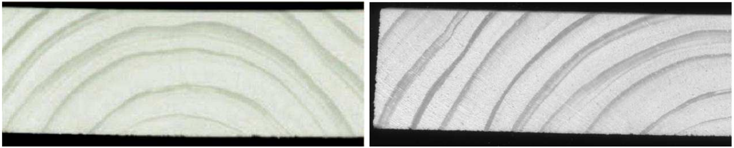
Mesh generation

Geometric description of Porous Medium - *MeshPore*[3],

Boards



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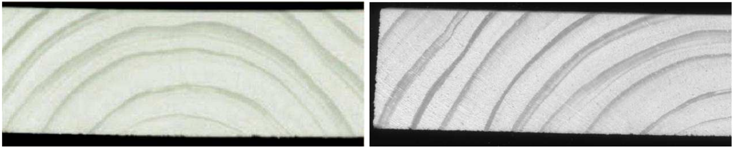

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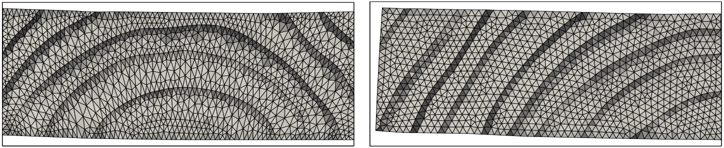

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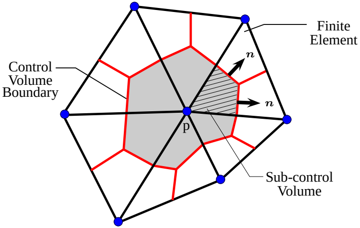
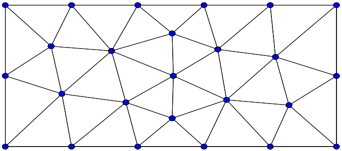
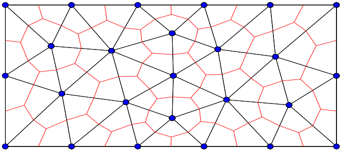
Spatial discretisation

Control-Volume Finite-Element (CV-FE) method





Triangular Mesh



Control Volume Mesh

Conservation equation

Liquid, Air and Energy conservation laws:

$$\frac{\partial \psi}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{q} = 0$$

Discrete conservation equations

For each control volume p in the mesh:

$$\frac{d\psi_p}{dt} + \frac{1}{V_p} \sum_{j=1}^{\mathrm{Nf}_p} (\boldsymbol{q} \cdot \boldsymbol{n})_j A_j \approx 0, \qquad \text{with } \mathrm{Nf}_p \text{ boundary faces.}$$

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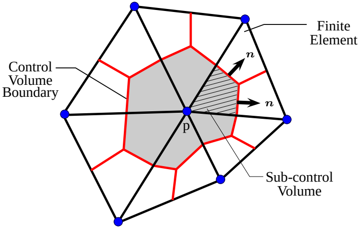

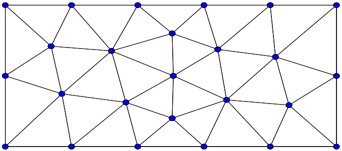

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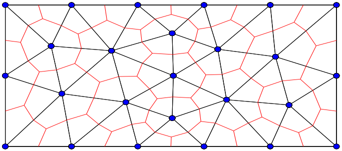

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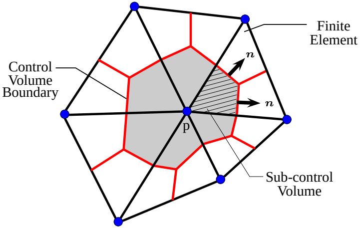

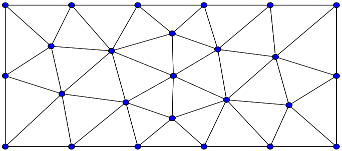

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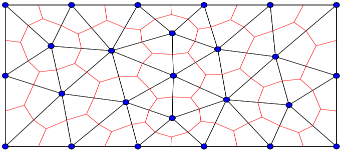

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Time integration

Jacobian-free Exponential Euler Method (EEM)

Advancement of drying system achieved using second-order accurate time-stepping formula[4]:

$$\frac{d\boldsymbol{u}}{dt} = \mathbf{g}(\boldsymbol{u}); \qquad \boldsymbol{u}(0) = \boldsymbol{u}_0$$

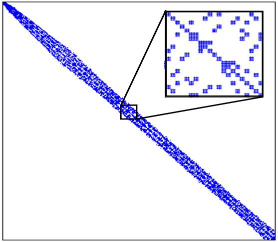
$$\frac{d\boldsymbol{u}}{dt} \approx \mathbf{g}(\boldsymbol{u}_n) + \mathbf{J}(\boldsymbol{u}_n)(\boldsymbol{u} - \boldsymbol{u}_n)$$

$$\boldsymbol{u}_{n+1} \approx \boldsymbol{u}_n + \mathbf{J}(\boldsymbol{u}_n)^{-1}(e^{\delta t \mathbf{J}(\boldsymbol{u}_n)} - \mathbf{I})\mathbf{g}(\boldsymbol{u}_n)$$

- Krylov subspace methods[4] used to effectively approximate the matrix-function vector product:

  $$\varphi(\boldsymbol{A})\boldsymbol{b} = \boldsymbol{A}^{-1}(e^{\boldsymbol{A}} - \mathbf{I})\boldsymbol{b}; \qquad \boldsymbol{A} \in \mathbb{R}^{N\times N}$$

- Our implementation[5] results in a speed-up of between 20%-40% on classical Newton approaches



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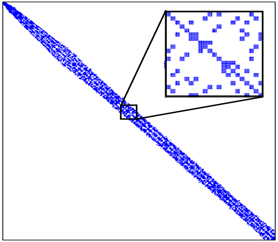

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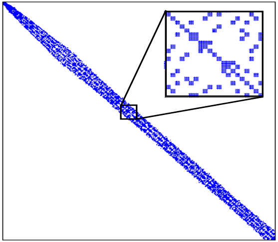

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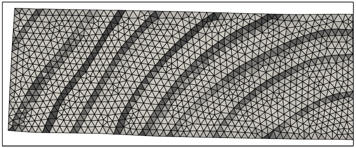
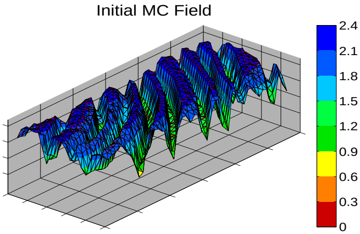
Low temperature drying

Temperature (Initial)	30	$(^\circ C)$
Temperature (Wet-bulb)	45	$(^\circ C)$
Temperature (Dry-bulb)	60	$(^\circ C)$
Average Moisture (Initial)	170	(%)
Air velocity	2	(ms^{-1})
Heat transfer coefficient	15	$(\mathrm{Wm}^{-2}\mathrm{K}^{-1})$
Mass transfer coefficient	0.015	(ms^{-1})

Quartersawn board



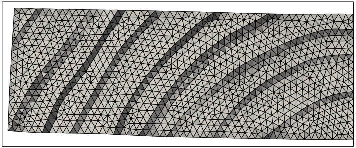
Virtual board



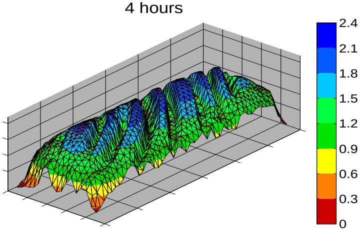
Low temperature drying

Temperature (Initial)	30	$(^\circ C)$
Temperature (Wet-bulb)	45	$(^\circ C)$
Temperature (Dry-bulb)	60	$(^\circ C)$
Average Moisture (Initial)	170	(%)
Air velocity	2	(ms^{-1})
Heat transfer coefficient	15	$(\mathrm{Wm}^{-2}\mathrm{K}^{-1})$
Mass transfer coefficient	0.015	(ms^{-1})

Quartersawn board



Virtual board



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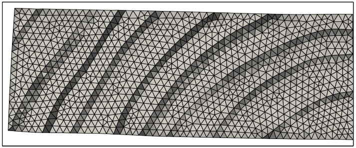
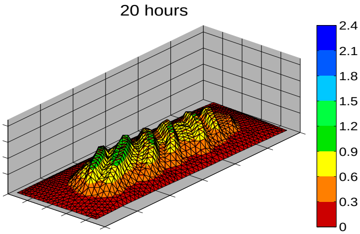
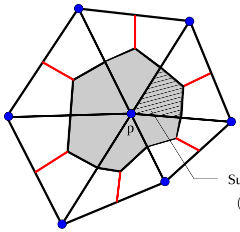


Virtual board



Formulation



Existing formulation of TransPore

Interpolate at each node in the mesh using a area-weighted average of the NSCV_p surrounding densities:

$$\rho_0(\boldsymbol{x}_p) = \frac{\displaystyle\sum_{i=1}^{\mathsf{NSCV}_p} \rho_0^{(i)} V_p^{(i)}}{\displaystyle\sum_{i=1}^{\mathsf{NSCV}_p} V_p^{(i)}}$$

Mass-conservative formulation

Partition the moisture content across the sub-control volumes ($X_p > X_{\rm fsp}$):

$$P_{\rm c}(\rho_0^{(i)}, S_{\rm w}^{(i)}, T_p) = P_{\rm c}^{\rm eqm}; \qquad \frac{\displaystyle\sum_{i=1}^{\mathsf{NSCV}_p} \rho_{\rm w}\phi^{(i)} S_{\rm w}^{(i)} V_p^{(i)}}{\displaystyle\sum_{i=1}^{\mathsf{NSCV}_p} \rho_0^{(i)} V_p^{(i)}} = X_p - X_{\rm fsp}$$

Ensures calibration of the driving forces, $P_{\rm w}$ and $P_{\rm g}$, within each control volume.

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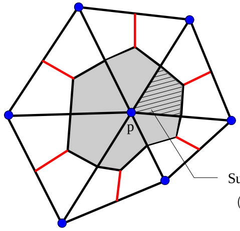

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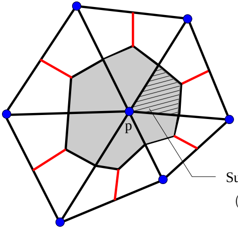

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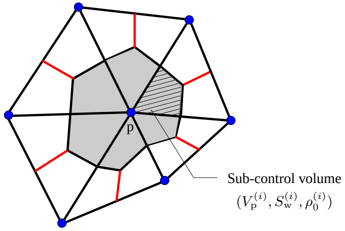

Mass-conservative formulation

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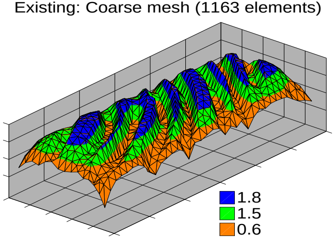
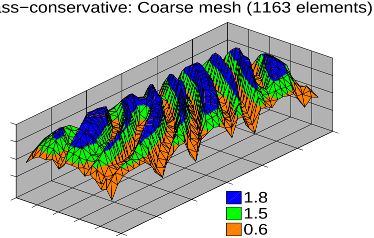
Results


Existing: Coarse mesh (1163 elements)
1.8
1.5
0.6


Mass−conservative: Coarse mesh (1163 elements)
1.8
1.5
0.6

Results

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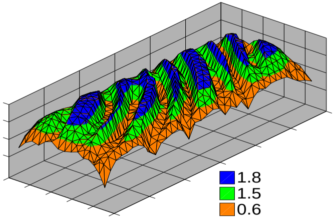
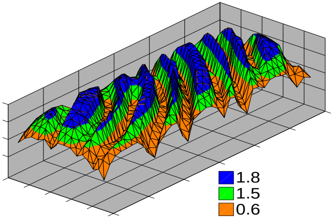
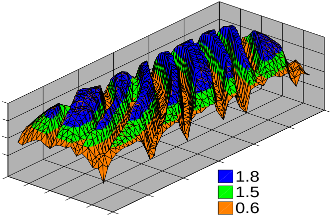


Mass−conservative: Coarse mesh (1163 elements)



Fine Mesh (2068 elements)



Summary

Two improvements to the mesoscopic modelling approach:

1. **Jacobian-free Exponential Euler Method**
   - Avoids forming the Jacobian
   - Results in a 20%-40% improvement in simulation times
2. **Mass-conservative formulation**
   - Better accounted for the rapid spatial variation within a growth ring
   - Allows for higher moisture peaks in earlywood and lower moisture zones in latewood to be correctly captured
   - Relaxes the requirement of using a very fine mesh as previously reported for the mesoscopic modelling approach[6]

[6] P. Perré and I. W. Turner (2008)

Acknowledgements

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Queensland University of Technology
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ENGREF
INSTITUT DES SCIENCES ET INDUSTRIES DU VIVANT ET DE L'ENVIRONNEMENT
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Thank you for listening!