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Novel calculation of thermal diffusivity from laser flash experiments

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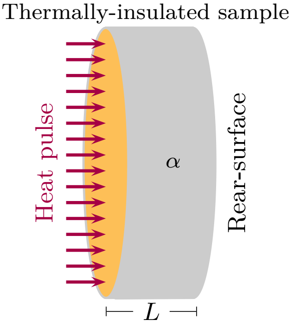
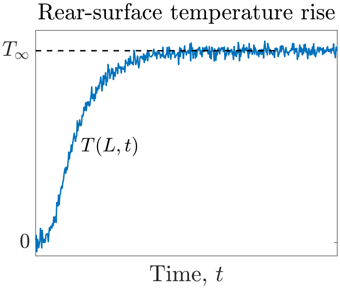
Most popular (Vozár and Hohenauer, 2003) and standard method (ASTM E1461-13, 2013) for measuring the thermal diffusivity of solids.



α – thermal diffusivity



- Governing equations (Parker et al., 1961):

$$\frac{\partial T}{\partial t}(x,t) = \alpha \frac{\partial^2 T}{\partial x^2}(x,t), \quad 0 < x < L, \quad t > 0,$$

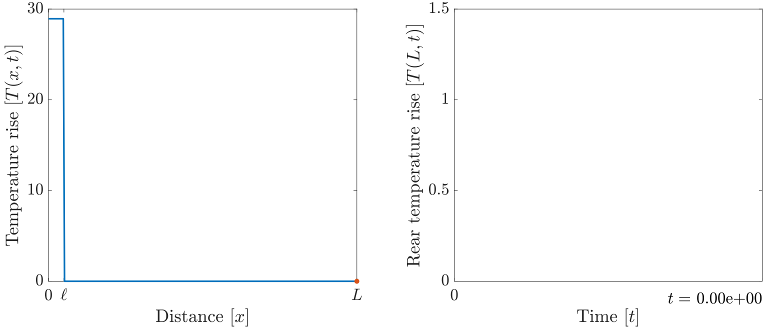
$$T(x,0) = \begin{cases} \dfrac{Q}{\rho c \ell} & 0 < x < \ell, \\ 0 & \ell < x < L, \end{cases}$$

$$\frac{\partial T}{\partial x}(0,t) = 0 \ \text{(front surface)}, \quad \frac{\partial T}{\partial x}(L,t) = 0 \ \text{(rear surface)}.$$

- Nomenclature:
  - $T(x,t)$: temperature rise above initial temperature at location x and time t [°C].
  - α: thermal diffusivity [$\mathrm{m}^2\,\mathrm{s}^{-1}$].
  - ℓ: depth at the front surface in which the heat pulse is instantaneously absorbed [m].
  - L: length of the sample [m].
  - Q: amount of heat absorbed through the front surface per unit area [$\mathrm{J\,m}^{-2}$].
  - ρc: volumetric heat capacity [$\mathrm{J\,K}^{-1}\,\mathrm{m}^{-3}$].

Half-rise time approach

Example spatial profile of temperature over time



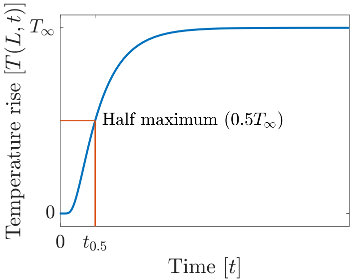
- Theoretical temperature rise at rear-surface:

$$T(L,t) = T_\infty\left[1 + 2\sum_{n=1}^{\infty}(-1)^n \frac{\sin(n\pi\ell/L)}{n\pi\ell/L} e^{-n^2\omega}\right],$$

where $\omega = \frac{\pi^2 \alpha t}{L^2}$ and $T_\infty = \frac{Q}{\rho c L}$.

- For small ℓ:

$$T(L,t) \approx T_\infty\left[1 + 2\sum_{n=1}^{\infty}(-1)^n e^{-n^2\omega}\right] =: \widetilde{T}(L,t).$$



- Since $\widetilde{T}(L,t) = 0.5T_\infty$ when $\omega = 1.370$ (displayed to four significant figures), we have:

$$\alpha \approx \frac{1.37L^2}{\pi^2 t_{0.5}},$$

where $t_{0.5}$ is the time required for the rear-surface temperature to reach $0.5T_\infty$.

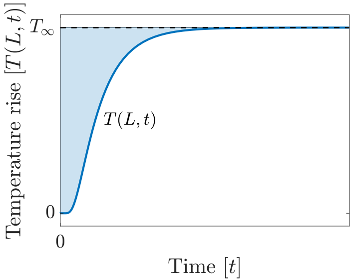
- Aim: obtain a closed-form expression for

$$\int_0^\infty [T_\infty - T(L,t)]\,\mathrm{d}t,$$

involving the thermal diffusivity, α.

- New formula for the thermal diffusivity:

$$\boxed{\alpha = \frac{T_\infty(L^2 - \ell^2)}{6\int_0^\infty [T_\infty - T(L,t)]\,\mathrm{d}t}}$$



- For equally-spaced discrete rear-surface temperature data ($\widetilde{T}_i$ for $i = 0,\ldots,N$):

$$\alpha \approx \frac{L^2}{6}\left[\sum_{i=1}^{N}\left(1 - \frac{\widetilde{T}_{i-1} + \widetilde{T}_i}{2T_\infty}\right)\Delta t_i\right]^{-1}.$$

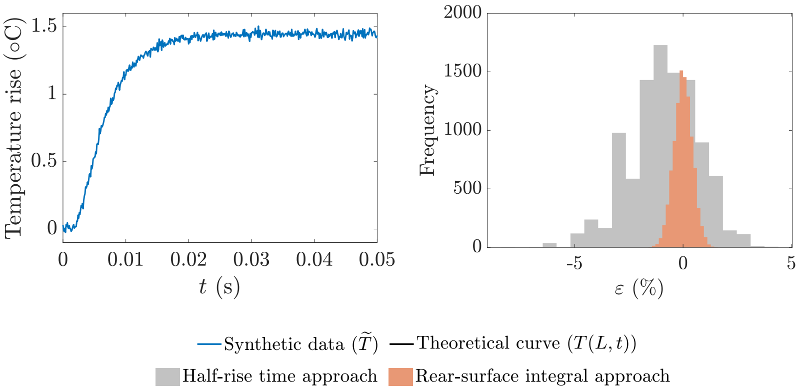
where Δt_i is the time between rear-surface temperature measurements.

Rear-surface integral approach

Carr (2019), *Chemical Engineering Science*

Moderate level of noise ($\sigma(z_i) = 0.02\,^\circ\mathrm{C}$)

Signed relative error: $\varepsilon = \dfrac{\alpha_{\text{exact}} - \alpha_{\text{estimate}}}{\alpha_{\text{exact}}}$



Christyn Wood

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AMSI Vacation Research Scholar (VRS) Project (2018–2019)

Project Objectives:

1. Remove non-physical assumption that heat is instantaneously absorbed at the front surface.
2. Extend to heterogeneous samples comprising two layers with different thermo-physical properties.

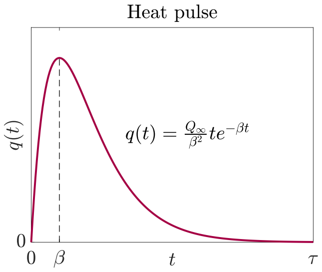
- Governing equations (Azumi and Takahashi, 1981; Czél et al., 2013; Heckman, 1973):

$$\frac{\partial T}{\partial t}(x,t) = \alpha\frac{\partial^2 T}{\partial x^2}(x,t), \quad 0 < x < L, \quad t > 0,$$

$$T(x,0) = 0, \quad 0 < x < L,$$

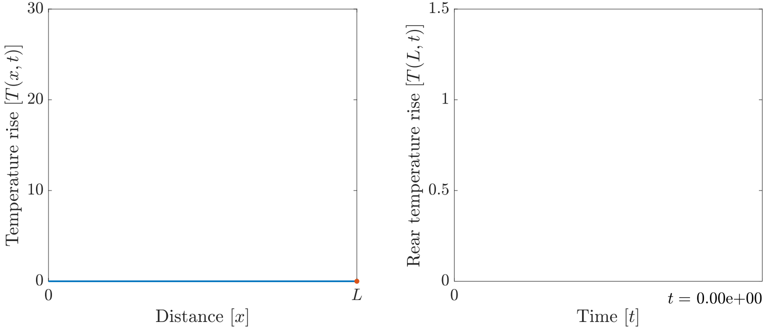
$$-k\frac{\partial T}{\partial x}(0,t) = q(t) \ \text{(front surface)}, \quad \frac{\partial T}{\partial x}(L,t) = 0 \ \text{(rear surface)}.$$

- Nomenclature:
  - $T(x,t)$: temperature rise above initial temperature at location x and time t [°C].
  - α: thermal diffusivity [m^2 s^{-1}].
  - k: thermal conductivity [W m^{-1} K^{-1}].
  - $q(t)$: heat flux applied by the laser pulse at the front surface [m].
  - L: length of the sample [m].



Rear-surface integral approach

Incorporating heat pulse

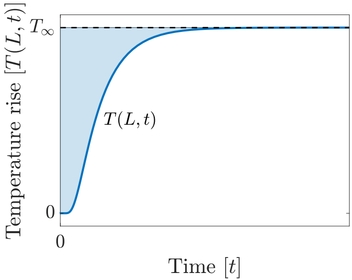


Rear-surface integral approach
Incorporating heat pulse

- Aim: obtain a closed-form expression for

$$\int_0^\infty [T_\infty - T(L,t)]\,\mathrm{d}t,$$

involving the thermal diffusivity, α.



- Note that $\int_0^\infty [T_\infty - T(L,t)]\,\mathrm{d}t = u(L)$, where

$$u(x) = \int_0^\infty [T_\infty - T(x,t)]\,\mathrm{d}t.$$

- Derive a differential equation satisfied by $u(x)$:

$$u''(x) = \int_0^\infty \underbrace{-\frac{\partial^2 T}{\partial x^2}\,\mathrm{d}t = \int_0^\infty -\frac{1}{\alpha}\frac{\partial T}{\partial t}}_{\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial x^2}}\,\mathrm{d}t = \frac{1}{\alpha}[T(x,0) - T_\infty] = -\frac{T_\infty}{\alpha}.$$

- Solution is a quadratic: $u(x) = c_0 + c_1 x - \dfrac{T_\infty x^2}{2\alpha}$

Rear-surface integral approach

Incorporating heat pulse

- Boundary and auxiliary conditions (where $Q_\infty = \int_0^\infty q(t)\,\mathrm{d}t$):

$$\text{Front surface}: \quad u'(0) = \int_0^\infty -\frac{\partial T}{\partial x}(0,t)\,\mathrm{d}t = \int_0^\infty \frac{q(t)}{k}\,\mathrm{d}t = \frac{Q_\infty}{k},$$

$$\text{Rear surface}: \quad u'(L) = \int_0^\infty -\frac{\partial T}{\partial x}(L,t)\,\mathrm{d}t = \int_0^\infty 0\,\mathrm{d}t = 0,$$

$$\text{Heat conservation}: \quad \rho c \int_0^L u(x)\,\mathrm{d}x = \int_0^\infty \left[Q_\infty - Q(t)\right]\,\mathrm{d}t.$$

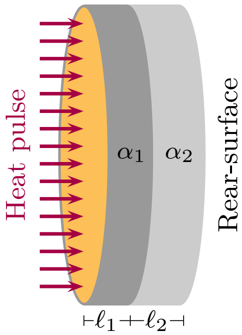
- Solution of boundary value problem (where $Q(t) = \int_0^t q(s)\,\mathrm{d}s$):

$$u(x) = \frac{\int_0^\infty \left[Q_\infty - Q(t)\right]\,\mathrm{d}t}{\rho c L} + \frac{T_\infty L}{\alpha}\left[x - \frac{L}{3} - \frac{x^2}{2L}\right].$$

- Recalling $\int_0^\infty \left[T_\infty - T(L,t)\right]\,\mathrm{d}t = u(L)$ yields thermal diffusivity:

$$\boxed{\alpha = \frac{L^2}{6}\left\{\int_0^\infty \left[1 - \frac{T(L,t)}{T_\infty}\right]\mathrm{d}t - \int_0^\infty \left[1 - \frac{Q(t)}{Q_\infty}\right]\mathrm{d}t\right\}^{-1}}$$

Rear-surface integral approach
Two-layer samples



$$\frac{\partial T_1}{\partial t}(x,t) = \alpha_1 \frac{\partial^2 T_1}{\partial x^2}(x,t), \quad 0 < x < \ell_1, \quad t > 0,$$
$$\frac{\partial T_2}{\partial t}(x,t) = \alpha_2 \frac{\partial^2 T_2}{\partial x^2}(x,t), \quad \ell_1 < x < L, \quad t > 0,$$

Initial conditions:

$$T_1(x,0) = 0, \quad 0 < x < \ell_1,$$
$$T_2(x,0) = 0, \quad \ell_1 < x < L,$$

Boundary conditions:

$$-k_1 \frac{\partial T_1}{\partial x}(0,t) = q(t), \quad t > 0,$$
$$\frac{\partial T_2}{\partial x}(L,t) = 0, \quad t > 0,$$

Interface conditions:

$$T_1(\ell_1,t) = T_2(\ell_1,t), \quad t > 0,$$
$$k_1 \frac{\partial T_1}{\partial x}(\ell_1,t) = k_2 \frac{\partial T_2}{\partial x}(\ell_1,t), \quad t > 0.$$

- Similar analysis yields the thermal diffusivity in each layer:

$$\boxed{\begin{aligned} \alpha_1 &= \frac{\ell_1^2(\ell_1\rho_1c_1 + 3\ell_2\rho_2c_2)\alpha_2}{6\alpha_2(\rho_1c_1\ell_1 + \rho_2c_2\ell_2)\left(I_T - I_q\right) - \ell_2^2(3\ell_1\rho_1c_1 + \ell_2\rho_2c_2)} \\ \alpha_2 &= \frac{\ell_2^2(3\ell_1\rho_1c_1 + \ell_2\rho_2c_2)\alpha_1}{6\alpha_1(\rho_1c_1\ell_1 + \rho_2c_2\ell_2)(I_T - I_q) - \ell_1^2[\ell_1\rho_1c_1 + 3\ell_2\rho_2c_2]} \end{aligned}}$$

where

$$I_T = \int_0^\infty \left[1 - \frac{T_2(L,t)}{T_\infty}\right] \mathrm{d}t,$$

$$I_q = \int_0^\infty \left[1 - \frac{Q(t)}{Q_\infty}\right] \mathrm{d}t.$$

- Formulas express the thermal diffusivity in each layer in terms of the other layer.
- Level of accuracy is similar to homogeneous (single-layer) estimate.

Summary and Conclusions

Carr and Wood (2019)

- New formulas for calculating thermal diffusivity from laser flash experiments:
  - Remove assumption that heat is instantaneously absorbed at front surface.
  - Accommodate arbitrary heat pulse shapes and two-layer samples.
  - Accuracy and variability of estimates is similar to original formula of Carr (2019).
- Limitations:
  - Analysis is one-dimensional.
  - Material is assumed to be perfectly thermally insulated (no heat loss).

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Rear-surface integral method for calculating thermal diffusivity: Finite pulse time correction and two-layer samples

Check for updates

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Rear-surface integral approach

Incorporating heat pulse