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Calculating thermal diffusivity from laser flash experiments

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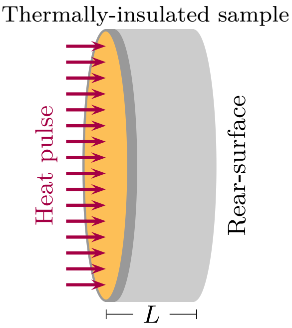
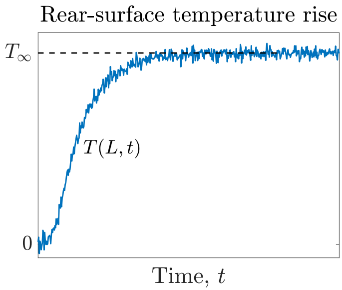
Most popular (Vozár and Hohenauer, 2003) and standard method (ASTM E1461-13, 2013) for measuring the thermal diffusivity of solids.





- Governing equations (Parker et al., 1961):

$$\frac{\partial T}{\partial t}(x,t) = \alpha \frac{\partial^2 T}{\partial x^2}(x,t), \quad 0 < x < L, \quad t > 0,$$

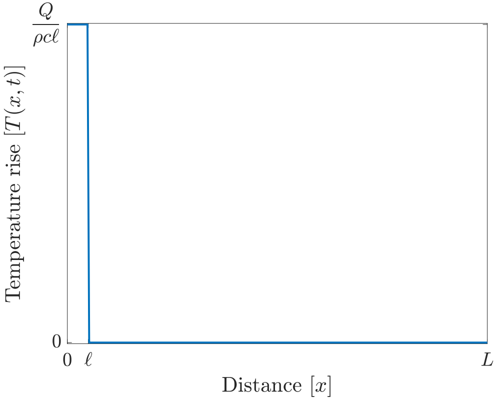
$$T(x,0) = \begin{cases} \dfrac{Q}{\rho c \ell} & 0 < x < \ell, \\ 0 & \ell < x < L, \end{cases}$$

$$\frac{\partial T}{\partial x}(0,t) = 0 \ \text{(front surface)}, \quad \frac{\partial T}{\partial x}(L,t) = 0 \ \text{(rear surface)}.$$

- Nomenclature:
  - $T(x,t)$: temperature rise above initial temperature at location x and time t [°C].
  - α: thermal diffusivity [$\mathrm{m^2\,s^{-1}}$].
  - ℓ: depth at the front surface in which the heat pulse is instantaneously absorbed [m].
  - L: length of the sample [m].
  - Q: amount of heat absorbed through the front surface per unit area [$\mathrm{J\,m^{-2}}$].
  - ρc: volumetric heat capacity [$\mathrm{J\,K^{-1}\,m^{-3}}$].

Transient solution

Example temperature distribution over time



Half-rise time approach
Formulation

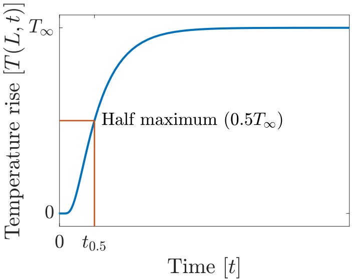
- Theoretical temperature rise at rear-surface:

$$T(L,t) = T_\infty\left[1 + 2\sum_{n=1}^{\infty}(-1)^n\frac{\sin(n\pi\ell/L)}{n\pi\ell/L}e^{-n^2\omega}\right],$$

where $\omega = \frac{\pi^2\alpha t}{L^2}$ and $T_\infty = \frac{Q}{\rho c L}$.

- For small ℓ:

$$T(L,t) \approx T_\infty\left[1 + 2\sum_{n=1}^{\infty}(-1)^n e^{-n^2\omega}\right] =: \widetilde{T}(L,t).$$



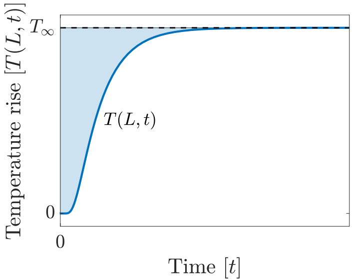
- Since $\widetilde{T}(L,t) = 0.5T_\infty$ when $\omega = 1.370$ (displayed to four significant figures), we have:

$$\alpha \approx \frac{1.37L^2}{\pi^2 t_{0.5}},$$

where $t_{0.5}$ is the time required for the rear-surface temperature to reach $0.5T_\infty$.

Rear-surface integral approach

Carr (2019)



- Idea: obtain a closed-form expression for

$$\int_0^\infty [T_\infty - T(L,t)]\,\mathrm{d}t,$$

involving the thermal diffusivity, α.

- Note that $\int_0^\infty [T_\infty - T(L,t)]\,\mathrm{d}t = u(L)$, where

$$u(x) = \int_0^\infty [T_\infty - T(x,t)]\,\mathrm{d}t.$$

- Derive a differential equation satisfied by $u(x)$:

$$u''(x) = \underbrace{\int_0^\infty -\frac{\partial^2 T}{\partial x^2}\,\mathrm{d}t = \int_0^\infty -\frac{1}{\alpha}\frac{\partial T}{\partial t}}_{\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial x^2}}\,\mathrm{d}t = \frac{1}{\alpha}\left[T(x,0) - T_\infty\right] = \begin{cases} \dfrac{T_\infty L}{\alpha}\left[\dfrac{1}{\ell} - \dfrac{1}{L}\right], & 0 < x < \ell, \\ -\dfrac{T_\infty}{\alpha}, & \ell < x < L. \end{cases}$$

Rear-surface integral approach

Carr (2019)

- Boundary conditions:

$$u'(0) = 0 \ \text{(front surface)}, \quad u'(L) = 0 \ \text{(rear surface)}.$$

- Interface conditions:

$$u(x) \text{ and } u'(x) \text{ are continuous at } x = \ell.$$

- Auxiliary condition:

$$\int_0^L u(x)\,\mathrm{d}x = 0.$$

- Solution of boundary value problem:

$$u(x) = \begin{cases} \dfrac{T_\infty L}{2\alpha}\left[\left(\dfrac{1}{\ell} - \dfrac{1}{L}\right)x^2 - \dfrac{1}{3}\left(2L - 3\ell + \dfrac{\ell^2}{L}\right)\right], & 0 \leq x \leq \ell, \\ \dfrac{T_\infty L}{2\alpha}\left[-\dfrac{1}{L}x^2 + 2x - \dfrac{1}{3}\left(2L + \dfrac{\ell^2}{L}\right)\right], & \ell \leq x \leq L. \end{cases}$$

Rear-surface integral approach
Carr (2019)

- Recalling $\int_0^\infty [T_\infty - T(L,t)]\,\mathrm{d}t = u(L)$ yields:

$$\int_0^\infty [T_\infty - T(L,t)]\,\mathrm{d}t = \frac{T_\infty(L^2-\ell^2)}{6\alpha},$$

- Rearrange to obtain the following formula for the thermal diffusivity

$$\alpha = \frac{T_\infty(L^2-\ell^2)}{6\int_0^\infty [T_\infty - T(L,t)]\,\mathrm{d}t}.$$

- For equally-spaced discrete rear-surface temperature data ($\widetilde{T}_i$ for $i = 0,\ldots,N$):

$$\alpha \approx \frac{(L^2-\ell^2)}{6}\left[\sum_{i=1}^{N}\left(1-\frac{\widetilde{T}_{i-1}+\widetilde{T}_i}{2T_\infty}\right)\Delta t_i\right]^{-1}.$$

where Δt_i is the time between rear-surface temperature measurements.

Numerical experiments
Synthetic experimental data

- Parameter values (Czél et al., 2013):

$$L = 0.002\,\mathrm{m}, \quad Q = 7000\,\mathrm{J\,m^{-2}}, \quad k = 222\,\mathrm{W\,m^{-1}K^{-1}},$$
$$\rho = 2700\,\mathrm{kg\,m^{-3}}, \quad c = 896\,\mathrm{J\,kg^{-1}K^{-1}}, \quad \ell = 0.0001\,\mathrm{m}.$$

- Target value of thermal diffusivity:

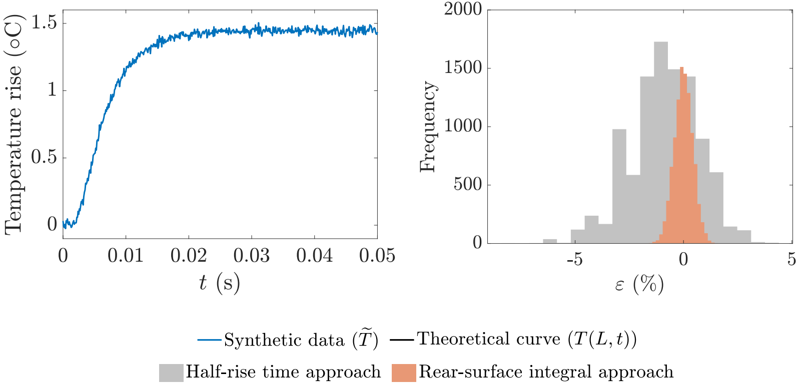
$$\alpha = \frac{k}{\rho c} = 9.1766 \times 10^{-5}\,\mathrm{m^2 s^{-1}}.$$

- Generate synthetic experimental data by adding Gaussian noise of mean zero to the theoretical rear-surface temperature rise curve:

$$\widetilde{T}_i = T(L, t_i) + z_i, \quad i = 0, \ldots, N, \quad t_i = \frac{i}{N} t_N.$$

- Use synthetic experimental data values, $\widetilde{T}_i$ for $i = 0, \ldots, N$, to calculate thermal diffusivity.

Moderate level of noise ($\sigma(z_i) = 0.02\,°C$)



Summary and Future Work

- Summary:
  - New formula for calculating thermal diffusivity from laser flash experiments:

$$\alpha = \frac{T_\infty(L^2-\ell^2)}{6\int_0^\infty [T_\infty - T(L,t)]\,\mathrm{d}t} \approx \frac{(L^2-\ell^2)}{6}\left[\sum_{i=1}^{N}\left(1-\frac{\widetilde{T}_{i-1}+\widetilde{T}_i}{2T_\infty}\right)\Delta t_i\right]^{-1}.$$

  - New formula produces estimates of the thermal diffusivity that are more accurate and less variable than the standard half-rise time approach.
- Future/current work (AMSI VRS Project - Christyn Wood): Remove not so realistic assumption that heat is instantaneously absorbed at the front surface.

Current IC and BCs

$$T(x,0) = \begin{cases} \dfrac{Q}{\rho c \ell} & 0 < x < \ell, \\ 0 & \ell < x < L, \end{cases}$$

$$\frac{\partial T}{\partial x}(0,t) = 0, \quad \frac{\partial T}{\partial x}(L,t) = 0.$$

New IC and BCs

$$T(x,0) = 0,$$

$$-k\frac{\partial T}{\partial x}(0,t) = q(t), \quad \frac{\partial T}{\partial x}(L,t) = 0.$$

Further reading

EJ Carr (2019), *Chemical Engineering Science*, in press.

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Rear-surface integral method for calculating thermal diffusivity from laser flash experiments

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ABSTRACT

The laser flash method for measuring thermal diffusivity of solids involves subjecting the front face of a small sample to a heat pulse of radiant energy and recording the resulting temperature rise on the opposite (rear) surface. For the adiabatic case, the widely-used standard approach estimates the thermal diffusivity from the rear-surface temperature rise history by calculating the half rise time: the time required for the temperature rise to reach one half of its maximum value. In this article, we develop a novel alternative approach by expressing the thermal diffusivity exactly in terms of the area enclosed by the rear-surface temperature rise curve and the steady-state temperature over time. Approximating this integral numerically leads to a simple formula for the thermal diffusivity involving the rear-surface temperature rise history. Using synthetic experimental data we demonstrate that the new formula produces estimates of the thermal diffusivity – for a typical test case – that are more accurate and less variable than the standard approach. The article concludes by briefly commenting on extension of the new method to account for heat losses from the sample.

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Numerical experiments
Additional Results

High level of noise ($\sigma(z_i) = 0.05\,^\circ$C)